

# **INVERSE MODELING OF OCEAN TIDES**

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## **LONG-TERM GOALS**

The principal long term goal of this project is to develop a practical, portable, nested, tidal data assimilation scheme which makes use of all available data (e.g., altimetry data, current moorings, coastal radar) to constrain barotropic and baroclinic tides (especially tidal currents) in coastal areas and shallow seas. A second long term goal is to develop more efficient methods for assimilation of very large oceanographic data sets.

## **OBJECTIVES**

We have previously developed an inverse approach to modeling global ocean tides which allows observational data (tide gauges, current meters, and satellite altimetry), and the hydrodynamic equations that the tidal fields must satisfy, to be combined rationally [Egbert, Bennett, and Foreman, 1994]. The present project is an outgrowth of this earlier work with several interrelated objectives: (1) To develop an improved global ocean tide solution (including estimates of uncertainties in the solution), (2) to explore the implications of the new solutions for global tidal energetics, (3) to adapt and improve the methods developed for the global model, so that they may be routinely applied to smaller scale (regional/coastal) barotropic tidal assimilation problems, and (4) to develop a better understanding of how uncertainties and approximations in the hydrodynamic equations limit the degree to which particular data types can constrain tidal currents and elevations in shallow water environments.

## **APPROACH**

Our general approach is to use a rigorous data assimilation scheme, based on minimizing a penalty functional defined in terms of weighted misfits to available tidal data, and the hydrodynamical equations. The weighting of data and dynamics is defined by rational estimates of errors in the two types of information. In our initial efforts we used simple dynamics (linear barotropic shallow water equations, with simple parameterizations for

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dissipation and tidal loading), but a rigorous and rather complicated assimilation scheme, using an approach based on explicit calculation of the representers of the data functionals [Bennett, 1992]. In this project we have retained the rigorous formulation of the generalized inverse approach, but developed significantly more efficient solution methods to allow for much larger data sets and more routine rapid application to smaller scale problems. We have also now generalized our linear inversion to allow for non-linearities in the dynamical equations, and modified our solution approach to make rapid application of the inversion scheme to small-scale regional and coastal tidal modeling problems practical.

## **WORK COMPLETED**

We have applied an iterative conjugate gradients approach to assimilate approximately  $10^6$  TOPEX/Poseidon crossover differences in an updated version of our global tidal solution (TPXO.3). With this new approach we solve the Euler-Lagrange equations in the data space, using the results of a coarse-grid, partial representer calculation for pre-conditioning (Egbert and Bennett, 1996; Bennett et al., 1996). The new global inverse solution has been used for studies of tidal energetics, and for testing consistency of altimetric estimates of open ocean tidal elevations with barotropic shallow water dynamics (Ray and Egbert, 1997; Egbert, 1997).

Programs for extracting tidal constants from along-track altimetry time series have been developed. This data has been used for construction of high-resolution (1/12 degree) inverse models for the Mediterranean Sea and the west coast of North America. For these solutions the full TOPEX data set (from the first 154 orbit cycles) were fit, with data from over 4000 spatial locations included in the Mediterranean sea solution. These regional tidal solutions have been the first test of our relocatable non-linear tidal inversion scheme. In this scheme we have allowed for non-linearities in the dynamical equations by linearizing in the neighborhood of the prior solution, and then doing a linear inversion. Current versions of all programs for extracting and inverting the altimetry data are general and relocatable, and thus allow for rapid construction of high resolution inverse tidal solutions in any regional sea.

Currently we are completing implementation of a significantly more efficient equation solver for the inverse solution. With the new approach the representer calculation is accomplished by direct factorization of a wave equation derived from the linearized shallow water equations. Once the equations are factored (this must be done only once for each tidal constituent) representer computations are very rapid. The factorization approach thus allows problems that previously required a week or more for solution on a small supercomputer (the OSU 64 node CM-5), to be run in a day or less on a high-end workstation (with approximately 500 Mb of memory). This new solution approach is especially advantageous for higher resolution models for which stable time stepping of the shallow water equations is very slow. As an initial test of this solution approach a 1/6 degree inverse solution is being constructed for the Indonesian archipelago/South China Sea.

Initial tidal analysis of a small coastal radar data set from the Oregon coast has been completed. Modifications of software to allow these estimates of surface tidal currents to be assimilated into a very high-resolution (1 km grid) barotropic model of coastal tidal currents

are underway. Initial theoretical work for development of a baroclinic inversion for coastal radar data has also begun.

## RESULTS

With the iterative solution scheme we have for the first time fit all T/P crossovers, demonstrating the feasibility of using a rigorous inversion approach with very large data sets. Compared to the CSR3.0 model (currently used as the default correction on the T/P GDR) the new inverse solution (TPXO.3) significantly reduces RMS residual cross-over differences in the altimetry data (global RMS: 7.98 cm for TPXO.3 vs. 9.64 cm for CSR3.0). Improvements are particularly significant in shallow water (12.78 cm vs. 22.32 cm), and near areas of complex bathymetry (Egbert, 1997). Calculation of the global tidal energy balance for TPXO.3 implies that over elongated bathymetric features oriented perpendicular to tidal flows (e.g., mid-ocean ridges), energy dissipation is significantly enhanced, presumably due to conversion of barotropic tidal motions into baroclinic modes. For the M2 tide our results suggest that globally more than 0.5 TW of energy is dissipated in this manner.

The 1/12 degree non-linear inverse model of the Mediterranean Sea appears to be a significant improvement over the best previously available tidal elevation solution from this area (i.e., Tsimplis et al., 1995). The new solution provides a significantly better fit to validation tide gauges (4.5 cm rms misfit for M2 vs. 6.1 cm), and to tidal constants estimated from altimetry data at cross-over points (1.1 cm rms misfit vs. 2.3 cm). Comparison of the non-linear inversion results to a purely linear inverse solution show little difference for elevations, but relatively significant differences for currents in shallow water. Due to the limited number of current data available it is not completely clear which solution for tidal currents is most accurate.

## IMPACT/APPLICATIONS

Results from this project have impacts on science and technology and applications in several distinct ways. First, our global model (which includes barotropic currents as well as elevations) is significantly improved, especially in shallow seas, and can be used to provide better boundary conditions for a variety of regional and local scale modeling and prediction problems. Second, our work on global tidal energetics has possibly significant consequences for vertical mixing in the open ocean (Munk, 1997). The estimated energy input over mid-ocean ridges due to conversion to baroclinic modes is 0.5 TW--of the same order of magnitude as energy input to the ocean by the global winds (1 TW). If this result holds up to further scrutiny, tidal mixing effects should almost certainly be incorporated into ocean climate modeling efforts. Third, our relocatable inversion scheme will allow for rapid construction of high resolution regional models of barotropic tidal elevations, making full use of the available altimetry and tide gage data. Finally, some of the ideas developed in this project to make inversion of large data sets feasible have found application in other more complicated data assimilation problems, such as the atmospheric general circulation inversion described by Bennett et al., [1996].

## TRANSITIONS

The global barotropic tidal solution is freely available via ftp (ftp.oce.orst.edu in pub/tides), and has been used by a number of researchers in a range of fundamental and applied studies. These include: detiding of altimetric time series such as TOPEX/POSEIDON; validating acoustic tomography data (Dushaw et al., 1997); removing tidal currents from the ATOC data (D. Cartwright, personal communication 1996); providing boundary conditions for applied environmental coast modeling (Blumberg, personal communication 1996); and removing tidal loading effects from geodetic and absolute gravity measurements (e.g., Baker et al., 1996). The relocatable tidal inversion code will be made available to researchers at NRL by January, 1998. Within the coming year, the relocatable barotropic inversion package will be made freely available on the internet to any interested researchers.

## RELATED PROJECTS

A NSF funded project for studying global tidal dynamics using satellite altimetry data has recently begun (in collaboration with R. Ray, Hughes STX/GSFC). Future work on global tidal data assimilation will be conducted with this funding. Improvements to inversion methods and software will be to some extent transferable from this NSF project to ONR funded research on coastal tidal modeling, and vice-versa.

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